Name: _____

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2020 Math Packet for Incoming Honors Precalculus/Trigonometry Students

Due Date: First Day of School

Classroom Supplies Required:

Graphing Calculator - preferably TI-84 or similar
Notebook
Graph Paper for Home
*(*Some students use a graph paper notebook for the class)*
3-ring Binder
Pencils

The following packet contains topics and definitions that you will be required to know in order to succeed in Honors Precalculus/Trigonometry this year. You should be familiar with each of the concepts as all of these topics were discussed in either Algebra II or Geometry and will be used frequently throughout the year. **All problems that you are to complete are marked in bold**.

Section 1: Coordinates and Planes

Midpoints:

The midpoint of the interval with endpoints a and b is found by taking the average of the endpoints.

$$M = \frac{a+b}{2}$$

The midpoint of a segment with endpoints at (x_1, y_1) and (x_2, y_2) is found by taking the average of the two coordinate values.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

1) **Find the midpoint M of the segment with endpoints C (4, 4) and D (-2, 6).**

Distance:

The distance d between any two points x_1 and x_2 on a real number line is:

$d = |x_{1-} x_2| = \sqrt{(x_2 - x_1)^2}$

The distance d between any two points (x_1, y_1) and (x_2, y_2) on a Cartesian plane is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between each pair of points:

2) **(6, -2) and (-5, 12)**

3) **(5.8, -1) and (7, -4)**

Section 2: Lines

Lines:

Slope: $\frac{y_2-y_1}{x_2-x_1}$

Slope Intercept Form: $y = mx + b$

Standard Form: $ax + by = c$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Intercepts:

In order to find the x-intercept(s) of an equation, you have to set y equal to zero and solve the equation for x. In order to find the y-intercepts of an equation, you have to set x equal to zero and solve the equation for y.

Parallel Lines:

Two lines whose graphs have the same slope.

Perpendicular Lines:

Two lines whose graphs have opposite reciprocal slopes.

Find the slope of the lines passing through each set of points:

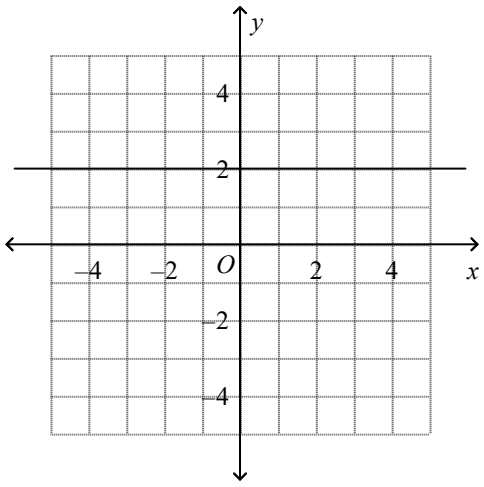
4) $(6, 12)$ ***and*** $(-6, -2)$

5) $\left(-\frac{1}{3}, 0\right)$ ***and*** $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

6) Find the slope of the line:

$3x + 5y = -15$

7) Find the slope of the following Line:



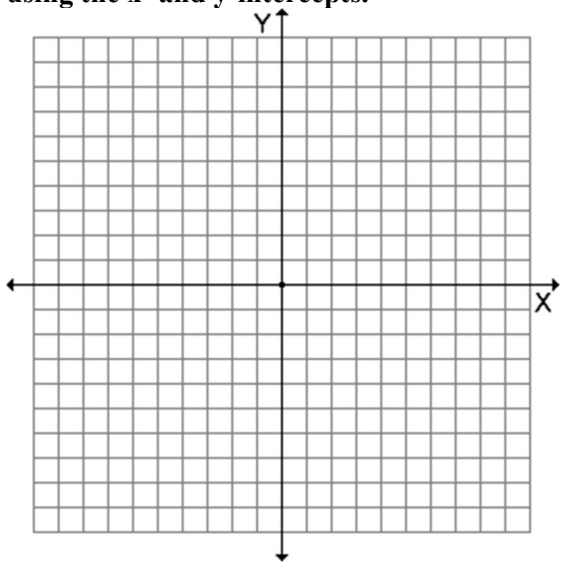
8) Write the equation of the line in standard form for a slope of -8 and through (-2, -2)

9) Find the point-slope form of the line through (-6, -4) and (2, -5).

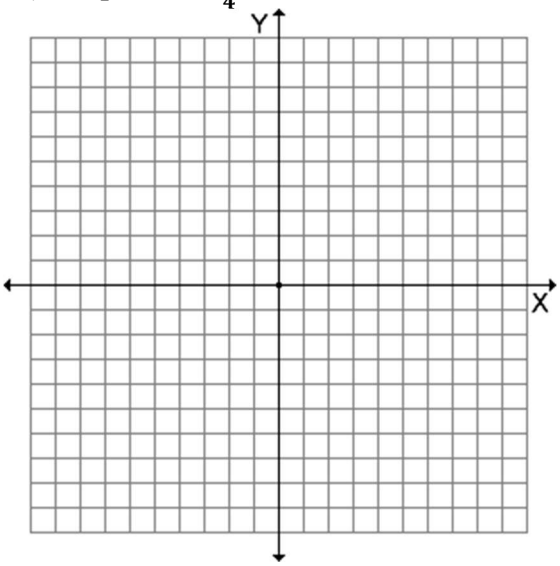
10) Write the equation of the line in slope-intercept form through $(-10, -3)$ and $(-1, 1)$.

11) Write the equation of the horizontal line that passes through $(-3, -1)$.

12) Graph the equation of 6x + 5y = 30 using the x- and y-intercepts.



13) Graph $y = -\frac{3}{4}x - 2$



14) Find the equation of the line that is parallel to $y = -3x + 4$ and goes through $(-4, 6)$.

15) Find the equation of the line perpendicular to $y = -\frac{5}{4}x + 1$ and goes through (2, 6).

16) Give the slope-intercept form for the equation of the line that is perpendicular to $7x + 3y = 18$ and passes through the point $(6, 8)$.

17) Show the work to determine which two lines are parallel?

I. $5y = -3x - 5$

II. $5y = -1 - 3x$

III. $3y - 2x = -1$

Section 3: Functions

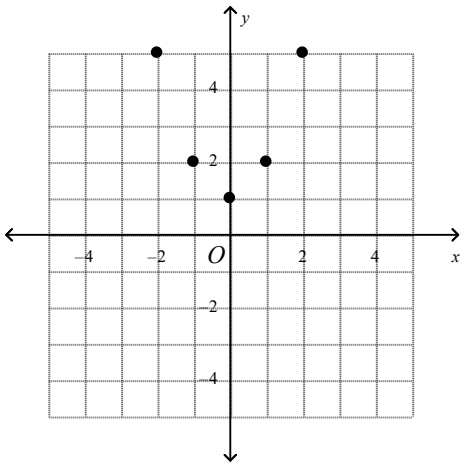
Function:

A function is a relation in which each value for the independent variable corresponds to a unique value of the dependent variable. A vertical-line test can be used on the graph of a relation in order to determine if the relation is a function. If a vertical-line can be drawn on the graph of a relation so that the line intersects 2 points on the graph, then the relation is not a function.

Domain: A list of all possible values for the independent variable.

Range: A list of all possible values for the dependent variable.

18) Write the ordered pairs for the relation. Find the domain and range and determine whether the relation is a function.



19) Suppose $f(x) = 4x - 2$ and $g(x) = -2x + 1$. Find the value of $\frac{f(5)}{g(-3)}$.

20) Evaluate each of the following for the function $f(x) = x^2 - 4x + 7$:

$f(4) =$

$f(-3) =$

21) Given that $f(x) = x^2 - 4$ $and \;\; g(x) = x + 2$	**Show the work to find:** $f(g(x)) =$
	Show the work to find: $g(f(x)) =$

22) Suppose $f(x) = 4x - 2$ and $g(x) = -2x + 1$. Find the value of $\frac{f(5)}{g(-3)}$.

23) Evaluate each of the following for the function $f(x) = x^2 - 4x + 7$:

$f(4) =$

$f(x + 2) =$

24) Evaluate the following combinations of functions. State the *domain* of the combined functions.

Let $f(x) = x - 1$ and $g(x) = x^2 + 3x - 4$

$f(x) \cdot g(x)$

$f(x) + g(x)$

$f(x) - g(x)$

$\frac{f(x)}{g(x)}$

Inverse Functions:

In order to calculate an inverse of a function algebraically, you must switch all of the x and y variables and solve the new equation for y. *The inverse only exists if the resulting equation is a function.*

Find the inverse (if it exists) of each of the following functions:

25) $\boldsymbol{f(x) = 3x + 2}$

26) $\boldsymbol{f(x) = 2x^2 - 4}$

27) $\boldsymbol{f(x) = \sqrt{x + 1}}$

Transformations:

Vertical Translations:	$y = f(x) \pm c$
Horizontal Translations:	$y = f(x \pm c)$
Y-axis flip:	$y = f(-x)$
X-axis flip:	$y = -f(x)$

Describe each of the following transformations:

28) $\boldsymbol{f(x) = -x^2 + 4}$

29) $\boldsymbol{f(x) = |-x| - 4}$

30) $\boldsymbol{f(x) = \sqrt{x - 3}}$

Zeros:

The zeros of a function are found by setting the function equal to zero and solving for each value of x. In the case of a rational function, you only need to set the numerator equal to zero and solve (provided the solutions do not also make the denominator equal to zero).

Show the work to find the zeros of each of the following functions:

31) $\boldsymbol{f(x) = 7x + 14}$

32) $\boldsymbol{f(x) = x^2 - 3x - 18}$

33) $\boldsymbol{f(x) = \frac{2x^2+4x}{x-2}}$

Inequalities: Inequalities are algebraic statements that contain <, >, $\leq$, or $\geq$.

Show the work to solve each inequality and graph its solution on a number line.

34) $\mathbf{7 + 6x > 19}$

35) $\mathbf{2(x + 2) - 3x \leq -1}$

36) $\mathbf{-3(4 - x) \geq 2(4x - 14)}$

Show the work to solve each system of equations. Check your solutions in both equations.

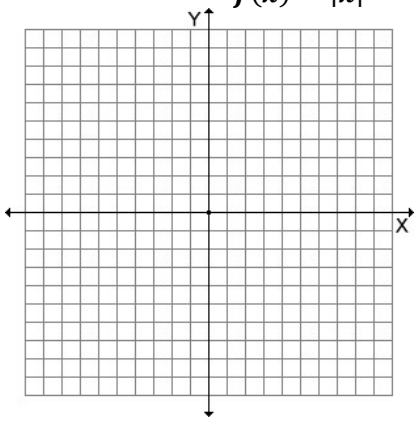
37) $y = 2x - 3$
$y = x - 1$

38) $3x + 2x = 4$
$y = 6x - 7$

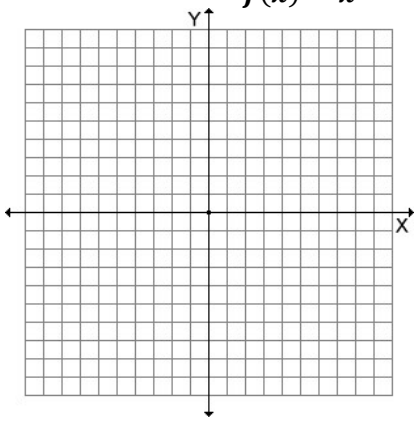
39) $5x - 6y = -32$
$3x + 6y = 48$

40) Sketch the graph of each functions:
Use a convenient scale for the x- and y-axis to make your graph large enough to see.

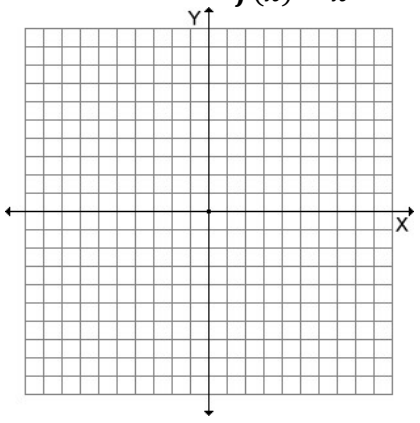
$f(x) = |x|$



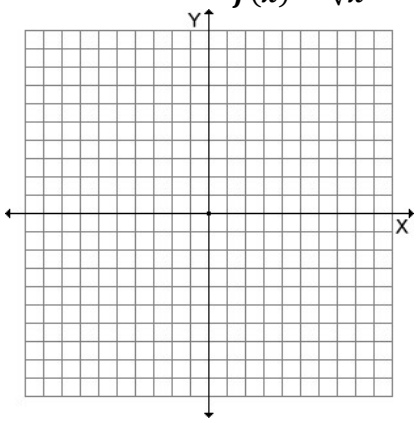
$f(x) = x$



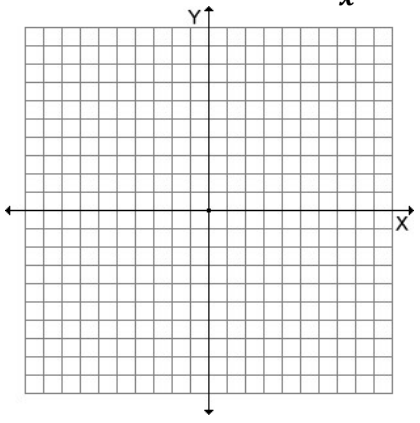
$f(x) = x^2$



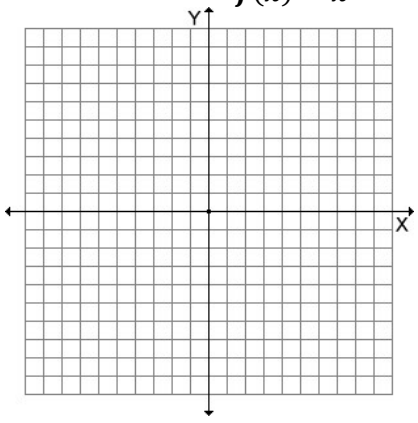
$f(x) = \sqrt{x}$



$f(x) = \frac{1}{x}$



$f(x) = x^3$



Section 4: Polynomials

Properties of Exponents:

1. Whole number exponents: $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ (n factors of x)
2. Zero exponents: $x^0 = 1, x \neq 0$
3. Negative Exponents: $x^{-n} = \frac{1}{x^n}$
4. Radicals (principal nth root): $\sqrt[n]{x} = a \rightarrow x = a^n$
5. Rational exponents: $x^{1/n} = \sqrt[n]{x}$
6. Rational exponents: $x^{m/n} = \sqrt[n]{x^m}$

Operations with Exponents:

1. Multiplying like bases: $x^n x^m = x^{m+n}$
2. Dividing like bases: $\frac{x^m}{x^n} = x^{m-n}$
3. Removing parentheses: $(xy)^n = x^n y^n$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $(x^n)^m = x^{nm}$

Simplify each of the following expressions:

41) $\mathbf{2a^2b^{-4} \cdot 4a^{-8}b^6}$

42) $\mathbf{\frac{8a^2b^{-2}}{4a^4c^{-5}}}$

43) $\mathbf{\left(\frac{3x^4}{y^{-2}}\right)^3}$

44) $\frac{2x^3+6x}{4x}$ **45)** $\frac{5x-4}{5x}$ **46)** $\frac{3xy^3+x^2y^2}{xy}$

Special Products:

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

Factor each of the following completely:

47) $\mathbf{4x^2 - 144}$

48) $\mathbf{8x^3 - 27}$

49) $\mathbf{64x^3 + 125y^3}$

50) $x^2 - 13x + 42$

51) $8x^2 - 10x - 3$

52) $4x^3 + 8x^2 - 5x - 10$

Binomial Theorem:

$$(x + a)^2 = x^2 + 2ax + a^2$$

$$(x - a)^2 = x^2 - 2ax + a^2$$

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$$

Expand each of the following:

53) $\boldsymbol{(x + 4)^2}$

54) $\boldsymbol{(x - 6)^2}$

55) $\boldsymbol{(x + 3)^3}$

56) $\boldsymbol{(x - 2)^3}$

57) Divide the following expression by <u>both</u> polynomial long division and synthetic division.

$$\frac{2x^3 - x^2 - 10x + 8}{x - 2}$$

Section V: Rational Expressions

Example: *Simplify the following expression* $\frac{3}{\frac{2}{x}+y}$

$\frac{3}{\frac{2}{x}+y*\frac{x}{x}}$ _____*multiply by* $\frac{x}{x}$ *to get a common denominator.*

$\frac{3}{\frac{2+yx}{x}}$ _____ *then add*

$3 * \frac{x}{2+yx}$ ----------*now multiply by the reciprocal*

$= \frac{3x}{2+xy}$ ----------*simplest form*

Show the steps to simplify each of the following expressions:

58) $\frac{\frac{1}{3}}{\frac{3}{b}}$ **59)** $\frac{\frac{3}{x-4}}{1-\frac{2}{x-4}}$ **60)** $\frac{\frac{2}{x}-5}{\frac{6}{x}-3}$

61) $\frac{2x}{x^2-1} - \frac{1}{x^2}$

62) $\frac{\sqrt{2a^7b^2}}{\sqrt{32b^3}}$

63) $\frac{\sqrt[3]{6x^2y^4}}{2\sqrt[3]{5x^7y}}$

Show the work to solve each of the following expressions:

64) $\frac{x}{5} = \frac{x+3}{8}$

65) $\frac{1}{x} + \frac{x}{2} = \frac{x+4}{2x}$

66) $\frac{x}{30} - \frac{1}{5x} = \frac{1}{6}$

Section V: Right Triangle Trigonometry

Special Right Triangles

45-45-90: In a 45-45-90 triangle, the side lengths have a ratio of $a : a : a\sqrt{2}$.

30-60-90: In a 30-60-90 triangle, the side lengths have a ratio of $a : a\sqrt{3} : 2a$

Trigonometry:

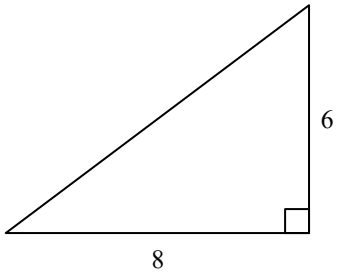
For any right triangle:

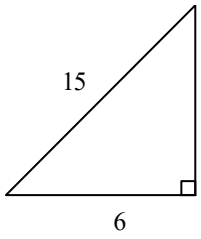
$$sin\theta = \frac{opposite\ side}{hypotenuse} \qquad cos\theta = \frac{adjacent\ side}{hypotenuse} \qquad tan\theta = \frac{opposite\ side}{adjacent\ side}$$

Pythagorean Theorem:

In a right triangle with legs a and b and hypotenuse c, then $a^2 + b^2 = c^2$.

67) Show the work to determine the length of the missing sides of the figures. Leave your answer in simplest radical form.





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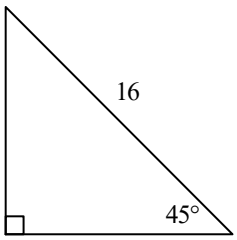

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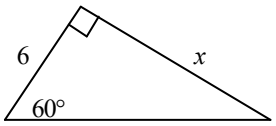

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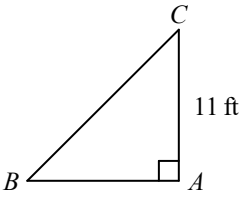
68) In triangle *ABC*, $\angle A$ is a right angle and $m\angle B$ = 45°. Find *BC*. If you answer is not an integer, leave it in simplest radical form. Find x and y in the other triangle.



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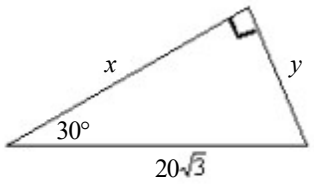

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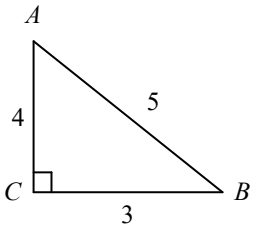
69) A piece of art is in the shape of an equilateral triangle with sides of 7 in. Show the work to find the area of the piece of art. Round your answer to the nearest tenth.

70) Write the ratios for sin *A*, cos *A, and* tan A.



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*Revised from Shaler Area School District