2021 Summer Math Packets for Incoming Geometry Honors Students

The math faculty at Shepaug Valley School would like to welcome you to 2021 - 2022 school year! We are looking forward to helping you achieve your greatest potential. We hope a quality education is one things you will value.

We have developed the attached review packet to help you prepare for the Honors Geometry class you will be taking this fall. This packet includes material that students are expected to understand before beginning the Honors Geometry curriculum. The topics covered by the packet are the foundational skills necessary to be successful in Honors Geometry. High School Geometry teachers will be collecting the packet and giving an assessment within the first few days of school. Please make sure to have this packet completed prior to the start of school. The summer assignment will be collected the first day of school.

Students may use any resources available to them to complete this packet. Helpful websites include:

www.purplemath.com
www.math.com
www.khanacademy.com

Please spend the time needed to do a quality job on this packet. Show and organize your work for each problem. Use a calculator ***where indicated*** but write down your calculations and show all of your work!

Enjoy your summer vacation and keep your education moving forward during this break.

Section 1: Simplifying Expressions & Solving Equations

Simplifying Expressions

Expressions are simplified when there is NO equal sign. Often times this involves combining like terms. Like terms are the same if and only if they have the same variable and degree.

Example 1:

$3 + 2y^2 - 7 - 5x - 4y^3 + 6x$

$-4y^3 + 2y^2 - 5x + 6x + 3 - 7$	Identify & reorder like terms to be together.
$-4y^3 + 2y^2 + 1x - 4$	Combine like terms.

Example 2:

$6a^2 - 2b + 4ab - 5a$ for $a = -3$ and $b = 4$

$6(-3)^2 - 2(4) + 4(-3)(4) - 5(-3)$	Substitute the values of a and b in the equation.
$6(9) - 8 + (-48) - (-15)$	Multiply.
$54 - 8 - 48 + 15$	Rewrite to avoid double signs.
13	Use order of operations to solve.

Solving Equations

When solving an equation, remember to combine like terms first. Take steps to isolate the variable by following the order of operations backwards and doing inverse operations.

Example 1:

$5k + 2(k + 1) = 23$

$5k + 2k + 2 = 23$	Distribute.
$7k + 2 = 23$	Combine like terms.
$7k = 21$	Subtract.
$k = 3$	Divide.

Example 2:

$10 - 4m = -5m + 3(m + 8)$

$10 - 4m = -5m + 3m + 24$	Distribute.
$10 - 4m = -2m + 24$	Combine like terms.
$-4m = -2m + 14$	Subtract.
$-2m = 14$	Add.
$m = -7$	Divide.

Section 1: Homework

Evaluate each expression.

1. $-(27 \div 9)$

2. $2[5^2 + (36 \div 6)]$

3. $\dfrac{5^2(4) - 5(4^2)}{5(4)}$

Evaluate each expression if *a* = 12, *b* = 9, and *c* = 4. Write your answer in simplest form. (Leave as an improper fraction.)

4. $4a + 2b - c^2$

5. $\dfrac{2c^3 - ab}{6}$

6. $2(a - b)^2 - 5c$

Solve each equation. Write your answer in simplest form. (Leave as an improper fraction.)

7. $30 = -4x - 6x$

8. $8x - 2 = -9 + 7x$

9. $12 = -4(-6x + 7)$

10. $-3(4x + 3) + 4(6x + 1) = 43$

11. $-5(12 - 3k) = -10(2 - 3k) + 5$

12. $7(-3y + 2) = 8(3y - 2)$

13. $5t + 5 = 3(5t - 4) - 10t$

14. $3(2b - 1) - 7 = 6b - 10$

15. $\dfrac{5x+1}{2} - 10 = 0$

16. $3x - 5 + \dfrac{x+1}{2} = 90$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 2: Polynomials

FOIL Method

The **FOIL** method is a special case of a more general method for multiplying algebraic expressions using the distributive property.

First	*First* terms of each binomial
Outer	*Outside* terms of each binomial
Inner	*Inside* terms of each binomial
Last	*Last* terms of each binomial

The general form is:

$$(a+b)(c+d) = \underbrace{ac}_{\text{first}} + \underbrace{ad}_{\text{outside}} + \underbrace{bc}_{\text{inside}} + \underbrace{bd}_{\text{last}}$$

$$\begin{aligned} (2y-7)(3y+5) &= (2y)(3y) + (2y)(5) + (-7)(3y) + (-7)(5) && \text{FOIL method} \\ &= 6y^2 + 10y - 21y - 35 && \text{Multiply.} \\ &= 6y^2 - 11y - 35 && \text{Combine like terms.} \end{aligned}$$

Example 1:

$(4t + 6)(2t – 8)$

$(4t)(2t) + (4t)(–8) + (6)(2t) + (6)(–8)$	FOIL method.
$8t^2 + –32t + 12t + (–48)$	Multiply.
$8t^2 – 20t – 48$	Combine like terms.

Factoring

Factoring is the process of "un-doing" a polynomial. Factors are numbers multiplied together to get a product.

Example 1:

$t^2 + 8t + 12$

1•12, 2•6, 3•4 are factors of 12	Identify the factors of the whole number.
2 and 6 can be added to get 8	Find the factors of 12 that add or subtract to equal 8.
$(t \quad + \quad)(t \quad + \quad)$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ – ____ + ____ = (–) (–) ____ – ____ – ____ = (–) (+) ____ + ____ – ____ = (–) (+)
$(t + 2)(t + 6)$	Substitute the numbers into the appropriate factors.

Example 2:

$x^2 - 6x + 8$

1•8, 2•4 are factors of 8	Identify the factors of the whole number.
2 and 4 can be added to get 6	Find the factors of 8 that add or subtract to equal 6.
$(x \quad - \quad)(x \quad - \quad)$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ − ____ + ____ = (−) (−) ____ − ____ − ____ = (−) (+) ____ + ____ − ____ = (−) (+)
$(x - 2)(x - 4)$	Substitute the numbers into the appropriate factors.

Example 3:

$p^2 - 3p - 40$

1•40, 2•20, 4•10, 5•8 are factors of 40	Identify the factors of the whole number.
5 and 8 can be subtracted to get 3	Find the factors of 40 that add or subtract to equal 3.
$(p \quad - \quad)(p \quad + \quad)$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ − ____ + ____ = (−) (−) ____ − ____ − ____ = (−) (+) ____ + ____ − ____ = (−) (+)
$(p - 8)(p + 5)$	Substitute the numbers into the appropriate factors.

Section 2: Homework

Find each product.

17. $(r + 1)(r - 2)$

18. $(n - 5)(n + 1)$

19. $(3c + 1)(c - 2)$

20. $(2x - 6)(x + 3)$

Factor each polynomial.

21. $p^2 + 9p + 20$

22. $g^2 - 7g + 10$

23. $n^2 + 3n - 18$

24. $y^2 - 5y - 6$

25. $t^2 + 9t - 10$

26. $r^2 + 4r - 12$

27. $d^2 - 12d + 27$

28. $y^2 - 2y - 24$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 3: Radicals

Radicals or roots are the "opposite" operation of applying exponents. You will undo exponents by using a radical.

Radical Symbol

Index

$$\sqrt[3]{\underbrace{x-4}_{\textbf{Radicand}}}$$

Read as the "cube root of x – 4"

Perfect Squares & Square Roots

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, ….

This list is the first 20 perfect squares. This means if you see any of these numbers under the radical you can quickly simplify it by finding the number that multiplies by itself to get the original number.

Example 1:

$\sqrt{144}$

$\sqrt{144} = \sqrt{12 \bullet 12} = 12$	Identify the number when multiplied by itself gives you the number under the radical.

$\sqrt{225}$

$\sqrt{225} = \sqrt{25 \bullet 25} = 25$	Identify the number when multiplied by itself gives you the number under the radical.

Non-Perfect Squares

When the number under the radical is not a perfect square, you have to reduce it to lowest terms.

Example 2:

$\sqrt{75}$

$\sqrt{75} = \sqrt{25} \bullet \sqrt{3}$	Identify the *largest* perfect square that divides evenly into the radical.
$5\sqrt{3}$	Take the square root of the perfect square radical and leave the non-perfect square under its radical.

$\sqrt{20}$

$\sqrt{20} = \sqrt{4} \bullet \sqrt{5}$	Identify the *largest* perfect square that divides evenly into the radical.
$2\sqrt{5}$	Take the square root of the perfect square radical and leave the non-perfect square under its radical.

$\sqrt{32}$

$\sqrt{32} = \sqrt{16} \bullet \sqrt{2}$	Identify the *largest* perfect square that divides evenly into the radical.
$4\sqrt{2}$	Take the square root of the perfect square radical and leave the non-perfect square under its radical.

$\sqrt{200}$

$\sqrt{200} = \sqrt{100} \bullet \sqrt{2}$	Identify the *largest* perfect square that divides evenly into the radical.
$10\sqrt{2}$	Take the square root of the perfect square radical and leave the non-perfect square under its radical.

Section 3: Homework

Simplify each radical expression.

29. $\sqrt{28}$

30. $\sqrt{54}$

31. $\sqrt{500}$

32. $\sqrt{72}$

33. $\sqrt{48}$

34. $\sqrt{150}$

35. $\sqrt{56}$

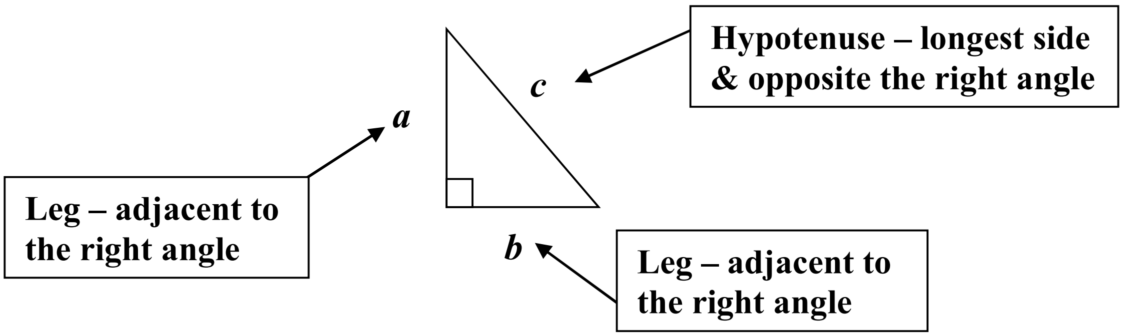
36. $\sqrt{27}$

37. $\sqrt{98}$

38. $\sqrt{120}$

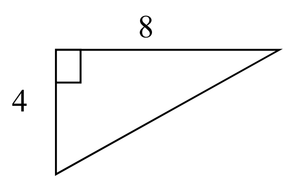
Section 4: Pythagorean Theorem

The Pythagorean Theorem is a formula unique to only right triangles.



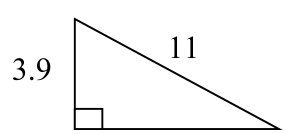
$$a^2 + b^2 = c^2$$

Example 1:



4 and 8 are the legs; hypotenuse is unknown	Identify the legs and the hypotenuse.
$4^2 + 8^2 = x^2$	Substitute the numbers into the equation.
$16 + 64 = x^2$ $80 = x^2$ $\sqrt{80} = \sqrt{x^2}$ $8.9 = x$	Solve using the rules of exponents and radicals. Round to the nearest tenth.

Example 2:

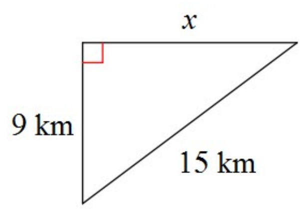


3.9 is a leg; 11 is the hypotenuse	Identify the legs and the hypotenuse.
$3.9^2 + x^2 = 11^2$	Substitute the numbers into the equation.
$15.21 + x^2 = 121$ $x^2 = 105.79$ $\sqrt{x^2} = \sqrt{105.79}$ $x = 10.3$	Solve using the rules of exponents and radicals. Round to the nearest tenth.

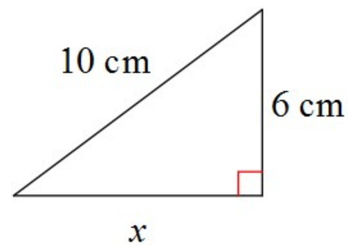
Section 4: Homework

Solve for the missing length using the Pythagorean Theorem. Round to the nearest tenth when necessary.

39.



40.



41. An architect is making a floor plan for a rectangular gymnasium. If the gymnasium is 24 meters long and 18 meters wide, what will the distance be between opposite corners? Draw a diagram and show all your work.

42. A ladder is leaning against the side of a 10 meter house. If the base of the ladder is 3 meters away from the house, how tall is the ladder? Draw a diagram and show all your work.

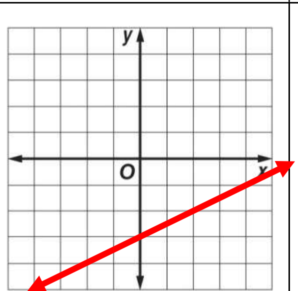
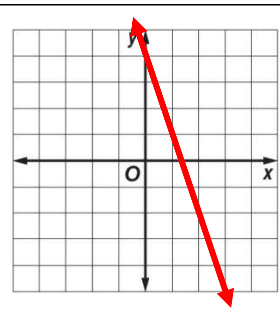
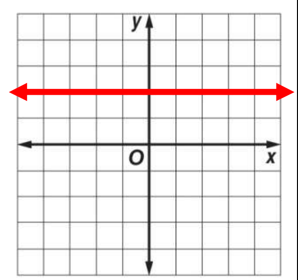
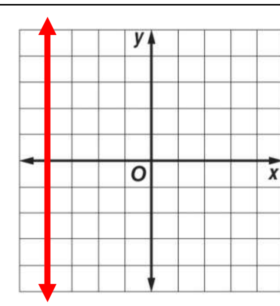
On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 5: Equations of Lines

Slope

Slope measures the steepness of a line. It is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run).

Equation: $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Positive	**Negative**	**Zero**	**Undefined**
m > 0	m < 0	$m = \dfrac{0}{\text{integer}} = 0$	$m = \dfrac{\text{integer}}{0} = \text{undef}$
		Horizontal Line	Vertical Line
$y = \dfrac{1}{2}x - 3$	$y = -3x + 4$	$y = 2$	$x = -4$
			

Example 1:

Calculate the slope between (2, –6) and (6, 4).

$m = \dfrac{4-(-6)}{6-2}$	Substitute the ordered pairs into the slope formula.
$m = \dfrac{4+6}{6-2} = \dfrac{10}{4} = \dfrac{5}{2}$	Simplify and reduce the fraction if necessary.

Writing Equations

You can use either the *slope-intercept form* or the *point-slope form* to write an equation of a line.

Slope-Intercept Form	Point-Slope Form	Standard Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$
m = slope b = y-intercept	m = slope (x_1, y_1) = point	slope = $-\dfrac{a}{b}$ A, B, C are integers $A \geq 0$

Example 2:

Write an equation of the line using the *slope-intercept form* that passes through the points (–1, 4) and (3, 12).

$m = \dfrac{12-4}{3-(-1)} = \dfrac{8}{4} = 2$	Calculate the slope.
$y = mx + b$ $12 = 2(3) + b$ $12 = 6 + b$ $b = 6$	Substitute one of the points (3, 12) and the slope (2) into the slope-intercept form and solve for *b*.
$y = 2x + 6$	Substitute *m* and *b* into the slope-intercept formula.

Example 3:

Write an equation of the line using the *point-slope form* that passes through the points (6, 7) and (3, 9).

$m=\frac{9-7}{3-6}=\frac{2}{-3}=-\frac{2}{3}$	Calculate the slope.
$y-y_1=m(x-x_1)$ $y-7=-\frac{2}{3}(x-6)$ $y-7=-\frac{2}{3}x+4$ $y=-\frac{2}{3}x+11$	Substitute one of the points (6, 7) and the slope $\left(-\frac{2}{3}\right)$ into the point-slope formula and solve for y.

Graphing Equations

When graphing linear equations you must be sure that the equation is in *slope-intercept form*.

$y=mx+b$

$m=slope$ and $b=y$–intercept

You start your graph at the y-intercept (where the graph crosses the y-axis.) Next you use your slope to go up or down and then to the right.

Use the following guidelines when plotting your slope.

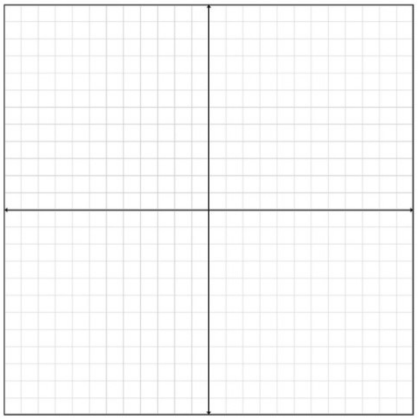
If the slope is **positive**, go **up** the value in the numerator. If it is **negative**, go **down** the value in the numerator.

Then ***always*** run to the **right** the denominator value.

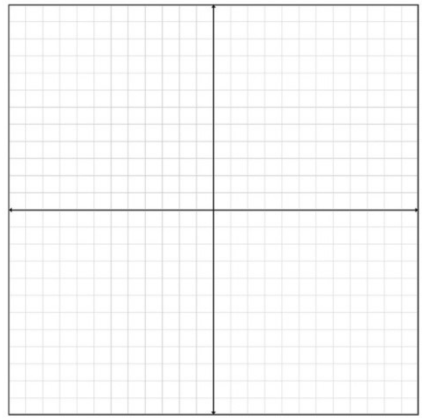
Section 5: Homework

For each set of ordered pairs, calculate the slope and write the equation of the line passing through each of the points in *slope-intercept form*. Then graph the equation.

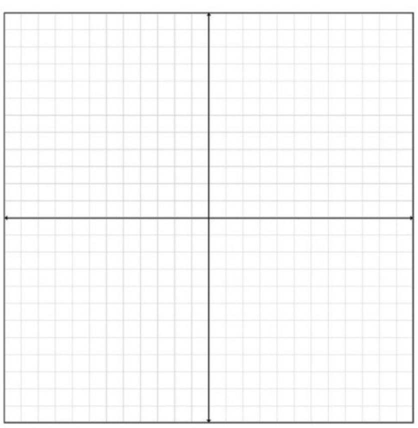
43. (0, –3) and (5, –1)



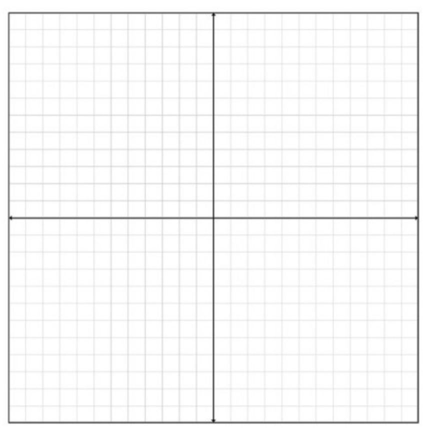
44. (4, 4) and (8, 3)



45. (5, –4) and (3, –4)



46. (9, –2) and (9, 4)



On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 6: System of Equations

A system of equations consists of having 2 or more equations. There are three methods to solving a system of equations: graphing, substitution, and elimination. We will only cover the last two in this review. When the lines intersect at exactly one point, the (x, y) values of that point are the solutions to the system.

Substitution

Example 1:

Solve for the following system of equations.

$$4x+3y=4$$
$$2x-y=7$$

$2x-y=7$ $-y=-2x+7$ $y=2x-7$	Solve for one of the variables in one of the equations.
$4x+3y=4$ $4x+3(2x-7)=4$ $4x+6x-21=4$ $10x-21=4$ $10x=25$ $x=2.5$	Substitute the expression for y into the other equation and solve for x.
$y=2x-7$ $y=2(2.5)-7$ $y=5-7$ $y=-2$	Substitute the value of x into either equation and solve for y.
$(2.5,-2)$	Express your answer as a point.

Elimination

Example 1:

Solve for the following system of equations.

$$3x + 7y = 15$$
$$5x + 2y = -4$$

$5[3x + 7y = 15]$ $-3[5x + 2y = -4]$	Eliminate either the *x* or *y* variables in both equations. Use the additive inverse property by multiplying the first equation by 5 and the second equation by –3 to eliminate the *x* terms.
$15x + 35y = 75$ $-15x - 6y = 12$	Add the two equations together.
$29y = 87$ $y = 3$	Solve for *y*.
$3x + 7(3) = 15$ $3x + 21 = 15$ $3x = -6$ $x = -2$	Substitute the value of *y* into either equation and solve for *x*.
$(-2,3)$	Express your answer as a point.

Section 6: Homework

Solve the following system of equations using substitution. (Express your answer as a point!)

47. $\begin{aligned} x+12y&=68 \\ x&=8y-12 \end{aligned}$

48. $\begin{aligned} 3x+2y&=6 \\ x-2y&=10 \end{aligned}$

Solve the following system of equations using elimination. (Express your answer as a point!)

49. $\begin{aligned} 2x+5y&=-4 \\ 3x-y&=11 \end{aligned}$

50. $\begin{aligned} 10x+6y&=0 \\ -7x+2y&=31 \end{aligned}$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5